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Occupational choice, incentives and wealth distribution ☆

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Abstract

We consider a model of occupational choice in large economies where individuals differ in their wealth endowment. Individuals can remain self-employed or engage in productive matches with another individual, i.e., form firms. Matches are subject to a moral hazard problem with limited liability. The division of the gains from such matches is determined by competitive forces. When the incentive problem is asymmetric, matches are typically wealth-heterogeneous, with richer individuals choosing the occupation for which incentives are more important. The utilities attained within a match depend on the wealth distribution and changes in the latter give rise to 'trickle down' effects.

JEL classification: C78; D31; D50; J41

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1. Introduction

How do incentives affect the relation between the distribution of wealth and the occupational decisions, matching patterns and payoffs that obtain in decentralized markets? We study this question for economies with asymmetric tasks in production and limited wealth transferability within matches.

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Our economies consist of a large number of risk-neutral individuals with different wealth endowments. Individuals may match together for production in contractual pairs that we will refer to as ‘firms’, or ‘partnerships’. These matches are subject to a moral hazard problem with limited liability (i.e., non-negative final wealth) constraints. Each partner can take one of two occupations (or ‘jobs’, or ‘tasks’, or ‘roles’) that differ in how the unverifiable component of effort affects the probability of success of the economic activity. Contracts within a match specify a distribution of the gains from production given an allocation of jobs to the parties, taking into account the parties’ reservation utilities or what they can earn in other matches and occupations. In equilibrium, the expected payoff for each partner in each occupation and match is equal to the reservation utility and the demand for jobs in the economy equals the supply.

Since roles have different productivities, equilibrium contracts should force incentives upon the individual in the more productive role. Surplus division then implies that this individual pays a transfer to the other party, and only *his* limited liability constraint can bind. When it does bind, there is a tension between maximization of the (incentive compatible) surplus and its division, and wealth effects arise. A threshold wealth level separates the individuals into two classes, with richer individuals choosing the occupation for which effort is more productive and incentives more important.

Wealth effects are pinned down by the wealth level of the poorest individual in the more productive job. The wealth level of this individual (corresponding to the median of the wealth distribution), affects the aggregate demand and supply of different occupations and determines the return to these occupations for each individual. Changes in the distribution that alter the median create trickle down effects by changing these returns. An increase in the median wealth makes rich individuals relatively abundant, making individuals above the median worse off and all those below better off.

Wealth effects are absent when, for example, all individuals are rich enough to afford the transfer that allows equal division of the maximum incentive compatible surplus. As a result, net payoffs are equalized across occupations and types, segregation or type-homogeneous matching is an equilibrium, and the wealth distribution does not matter for utilities. In the special case where the incentive problem is completely symmetric (i.e., occupations identical) no transfers are necessary to achieve equal division of the maximum surplus. Consequently, wealth effects never arise and segregation is always an equilibrium.

Our results depend on the interaction between the indivisibility of tasks, the related asymmetry in incentives across occupations, and two different wealth transferability problems. The first arises from the above-mentioned limited liability constraints on the ex-post division of the surplus. Non-convexities and inefficiencies due to limited liability and asymmetries can be eliminated if the individuals can commit to ex-ante surplus division schemes such as lotteries on occupations.[1] As long as participation constraints need bind only in expected terms over lottery

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[1] As in [14].

outcomes, any match can achieve the maximum incentive compatible surplus and use lotteries to divide it arbitrarily in ex-ante terms. In such a case, there would be no wealth effects. The second wealth transferability problem is created by the absence of such lotteries, i.e., by assuming that participation constraints must be satisfied for each possible outcome of a lottery on occupations.[2]

The asymmetry in incentives is a difference relative to other papers on matching markets where production or surplus are typically symmetric functions and matching is studied under other kinds of imperfections (see, e.g., [10,11] where credit market imperfections are considered; and [16] where matching occurs with frictions). Other models (e.g., [9,11,16,17]) explore the link between complementarities and segregation or monotonic matching. Complementarities generate monotone matching because richer individuals gain more from matching with the rich, even though high-wealth individuals are preferred by all. In our economies in contrast, only the wealth level of the individual in the more productive task can matter for surplus in equilibrium, and everyone has identical incentives of hiring a rich individual to undertake this task.

Our notion of equilibrium is linked to the f-core [6]. We make explicit the dependence of coalition payoffs on the outside option and exploit the reservation utility as an operational device to characterize equilibria. As in [16] and other matching models, the reservation utility plays the role of a price-like variable. Relative to the club theoretic literature (e.g., [3–5,13]), we essentially restrict the benefits of using lotteries on occupations and do not make their use explicit.[3] Other related papers on endogenous occupational choice are [1,7,8,12]. Our paper differs in either the moral hazard feature or the predictions about matching patterns.

2. The model

Primitives: The economy lasts two periods, time zero and one, and has no aggregate uncertainty. There are two physical commodities, consumption and leisure, and a continuum of individuals indexed by $i \in [0, 1]$. Individuals are all endowed with one unit of leisure but differ in their endowment of the consumption good, i.e., wealth. The verifiable wealth level (or type) of any individual i is an element of the finite set $\{W_k\}_{k \in \mathbf{K}}$, with $\mathbf{K} \equiv \{1, \ldots, K\}$. We assume that $W_k \geqslant 0$ for all k and $W_k > W_{k'}$ if $k > k'$. Let $\bar{\mu}_k > 0$ be the measure of individuals of type k with $\sum_{k \in \mathbf{K}} \bar{\mu}_k = 1$ and let $i_k \in [0, \bar{\mu}_k]$ be the ith individual of type k. All individuals are expected utility maximizers with identical Bernoulli utility functions given by $y - \frac{ce^2}{2}$, where $y \geqslant 0$ denotes final consumption, $e \in [0, 1]$ denotes time spent working, or

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[2] Such intermediate participation constraints arise if, as here, individuals are free to choose any occupation while they bargain. We are grateful to Patrick Legros and an anonymous referee for pointing out the role of lotteries. A complete analysis of lotteries goes beyond the scope of this paper.

[3] There are other differences between our model and club theoretic ones. For instance, [3] and others allow for exchange of capital and consumption goods across clubs, while we only focus on the labor market.

effort, and $c>1$ is a parameter representing the cost of effort. We denote an economy by $\theta = \{\bar{\mu}_k, W_k\}_{k \in \mathbf{K}}$, its distribution of wealth.

There is a production technology yielding a stochastic output X through the exercise of two tasks, p and a. We call the individual engaged in task a (respectively, task p) the a-agent (resp., the p-agent). Effort levels $e_a, e_p \in [0,1]$ exerted in tasks a and p affect the output X as follows:[4]

$$X = \begin{cases} X_2 & \text{with probability } f(e_p, e_a; \alpha), \\ X_1 & \text{with probability } 1 - f(e_p, e_a; \alpha), \end{cases}$$

where $X_2 > X_1 \geqslant 0$. We normalize $X_2 - X_1 = 1$ and assume that the probability f has the following form:

$$f(e_p, e_a; \alpha) = \alpha e_a + (1-\alpha) e_p,$$

where $\alpha \in [1/2, 1]$ is a parameter. The parameter α measures the degree of substitutability, defining the difference in occupation between being a p-agent or an a-agent. If $\alpha = 1/2$, then the occupations within the firm are completely symmetric. On the other hand, $\alpha = 1$ is the standard principal-agent setup where only the a-agent is asked to put some effort in. We assume that p- and a-agents contribute to the production of X also through other observable inputs, which makes it necessary to have participation of both agents regardless of α.

Markets: At time zero, frictionless matching markets are open. Each individual in the economy has the option to remain idle or to engage in the stochastic productive activity by choosing a match with an individual and one of the two occupations within the match. If individuals match, then bargaining on the division of the output takes place between the partners taking into account their wealth levels. The assumption of a large economy (a continuum of individuals) implies that bargaining within a match occurs taking as given what each can obtain in other matches and in other occupations. This is the competitive aspect of the matching market. At time one, negotiations are over, individuals receive their wealth endowments, production takes place, uncertainty gets resolved, output is distributed across partners and individuals consume their total income. There is no market for the exchange of output.

Bargaining and contracts: We assume that efforts exerted in a match are unobservable. The partners in a match thus have to specify the division of the surplus through a sharing rule $w = w(X)$ as a function of the total output X, given an allocation of tasks. When the realized output is X, the payment to the a-agent is $w(X)$ and the payment to the p-agent is $X - w(X)$. Due to risk-neutrality (i.e., the linearity of utility in final consumption), it is without loss of generality to assume that the a-agent commits his entire wealth W_a to the p-agent upfront, so that his *final* income is simply $w(X)$ whereas the p-agent's final income is $W_p + W_a + X - w(X)$. Thus, the expected utilities of final consumption net of the cost of effort for the

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[4] Extensions to the case where X takes more than two values can be accommodated with the usual monotonicity conditions.

partners in a match are given by

$$u_a(w, e_a, e_p) = E[w(X) \mid e_p, e_a] - \frac{ce_a^2}{2},$$

$$u_p(w, e_a, e_p, W) = W + E[X - w(X) \mid e_p, e_a] - \frac{ce_p^2}{2},$$

where $W \equiv W_p + W_a$. Notice that forming a match does not entail any set-up cost. A positive set-up cost may make it necessary for individuals to borrow in order to form a firm. However, such a possibility will not affect any of our qualitative results as long as the cost of borrowing is not type-dependent, i.e., financial markets are perfect.[5]

Given a sharing rule w, the effort chosen by each agent must be incentive compatible:

$$e_a \in \arg\max_{e' \in [0,1]} u_a(w, e', e_p), \qquad (\mathrm{IC}_a)$$

$$e_p \in \arg\max_{e' \in [0,1]} u_p(w, e_a, e', W). \quad (\mathrm{IC}_p)$$

The sharing rule w cannot force any individual to negative consumption in any state of the world. Non-negative consumption (or, $y \geqslant 0$) constrains the sharing rule w to satisfy the limited liability constraints

$$w(X) \geqslant 0, \qquad (\mathrm{LL}_a)$$

$$W + X - w(X) \geqslant 0 \quad (\mathrm{LL}_p)$$

for all X.

The limited liability and incentive compatibility constraints define a feasible set $\mathbf{F}(W)$ of expected utilities $U' \equiv (U_p, U_a)$:

$$\mathbf{F}(W) = \{U' \in \mathbb{R}^2 \mid \exists w, e_a, e_p, : (\mathrm{LL}), (\mathrm{IC}) \text{ hold}, \; U_p = u_p(w, e_a, e_p, W), \\ U_a = u_a(w, e_a, e_p)\}.$$

In the matching market, the utilities in $\mathbf{F}(W)$ that will be attained depend on the reservation utilities $\overline{U} = (\overline{U}_p, \overline{U}_a)$ that the two partners in a match can obtain from other matches and occupations. Successful bargaining within a match must guarantee each individual this reservation utility as switching to any other match is still possible throughout negotiations. Let $\mathbf{F}(W, \overline{U}) = \mathbf{F}(W) \cap \{U' \geqslant \overline{U}\}$ be the feasible set of utilities taking into account $\overline{U}$.

The utilities that will be attained within a match depend also on the bargaining procedure that will be employed within the match. A bargaining procedure $\mathscr{B}$ is a game form with a solution concept applied to it that, for each $W, \overline{U}$, picks a subset of pairs

$$U(\mathscr{B}, W, \overline{U}) \equiv (U_p(\mathscr{B}, W, \overline{U}), U_a(\mathscr{B}, W, \overline{U})) \subset \mathbf{F}(W, \overline{U})$$

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[5] The effect of introducing a positive set-up cost in the presence of financial market imperfections is discussed in [2].

if $\mathbf{F}(W, \overline{U})$ is non-empty, and picks (W_p, W_a) otherwise.[6] We consider bargaining procedures $\mathscr{B}$ that are constrained efficient. A bargaining procedure $\mathscr{B}$ is *constrained efficient* if for each $W, \overline{U}$, there does not exist $U' \in \mathbf{F}(W, \overline{U})$ such that $U' \geqslant U(\mathscr{B}, W, \overline{U})$, with at least one strict inequality.

Equilibrium: An individual in our economy has a choice among $2K + 1$ occupations: being a p-agent (denoted by P_k), an a-agent (denoted by A_k) of any of the K types of individuals, or being idle (denoted by I). Let $d_i \in \mathbf{D} \equiv \{\{P_{k'}\}_{k' \in \mathbf{K}}, \{A_{k'}\}_{k' \in \mathbf{K}}, I\}$ be the occupational decision of individual i. For each k, let

$$\mu_k(d) = Leb(\{i_k \in [0, \bar{\mu}_k] \mid d_{i_k} = d\})$$

be the Lebesgue measure of individuals of type k who make the occupational decision $d \in \mathbf{D}$. Let $\mu_k = \{\mu_k(d)\}_{d \in \mathbf{D}}$ be the vector of measures of individuals of type k who are in different occupations and let $\mu = \{\mu_k\}_{k \in \mathbf{K}}$ be an allocation for the economy θ. With the ordered pair (k, k') we denote the match where k is p- and k' is a-agent.

At time zero, in the decentralized matching market, individuals maximize their utility by choosing an occupation and a match, taking as given a vector of type-dependent reservation utilities $U = \{U_k\}_{k \in \mathbf{K}}$. Occupational patterns in equilibrium will endogenously determine the utility U_k for each type k. Given U, for each individual i_k, a decision $d \in \mathbf{D}$ yields utility

$$U_k(d, U) = \begin{cases} W_k & \text{if } d = I, \\ U_a(\mathscr{B}, W_{k'} + W_k, U_{k'}, U_k) & \text{if } d = A_{k'}, \\ U_p(\mathscr{B}, W_k + W_{k'}, U_k, U_{k'}) & \text{if } d = P_{k'}. \end{cases}$$

An equilibrium requires optimization, consistency of individual behavior with aggregate figures, and that there be no surplus over outside options.

Definition 1. An equilibrium for an economy θ with bargaining procedure $\mathscr{B}$ consists of a vector of individual decisions $\{d_{i_k}\}_{i_k, k}$, an allocation μ, and a vector of utilities U, such that

(i) (optimization) for all k and all $i_k \in [0, \bar{\mu}_k]$, given U,

$$d_{i_k} \in \arg\max_{d \in \mathbf{D}} U_k(d, U);$$

(ii) (market clearing) for all $k, k' \in \mathbf{K}$,

$$\mu_{k'}(P_k) = \mu_k(A_{k'}) \quad \text{and} \quad \sum_{d \in \mathbf{D}} \mu_k(d) = \bar{\mu}_k$$

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[6] The assumption that the solution is (W_p, W_a) when $\mathbf{F}(W, \overline{U})$ is empty is made to anchor the equilibrium U to an actual match, avoiding meaningless equilibria. While in general $U(\mathscr{B}, W, \overline{U})$ may depend on each wealth level separately, and not just on their sum W, this added generality will be seen to be immaterial, given our notion of equilibrium.

and,

(iii) (no extra surplus) for all k,

$$U_k = \max_{d \in \mathbf{D}} U_k(d, U).$$

Condition (i) states that each individual chooses the occupation that gives him the highest utility. Condition (ii) ensures market clearing and is the demand equal to supply equivalent in this discrete goods economy, avoiding the possibility that, say, a measure one half of type k individuals be matched to a measure one third of type k' individuals. Condition (iii) states that U must correspond to the utility obtained from the best occupational choice available to each individual, i.e., there is no surplus to bargain over in any match, in equilibrium. Note that it is an equation, not an identity.

Condition (iii) in the definition of equilibrium implies that the equilibrium outcome is invariant to the choice of constrained efficient bargaining procedures $\mathscr{B}$. To see this, suppose that under the constrained efficient procedure $\mathscr{B}$ the equilibrium is $\{d^*_{i_k}\}_{i_k,k}$, μ^*, U^*. By Definition 1(iii), in any match (k, k') (whether or not it forms under μ^*), the partners got either the utility pair $(U^*_k, U^*_{k'})$ or the pair $(W_k, W_{k'})$ under the procedure $\mathscr{B}$. Fixing μ^* and taking U^* as given, consider an alternative constrained efficient bargaining procedure $\mathscr{B}'$. Since $\mathbf{F}(W_k + W_{k'}, U^*_k, U^*_{k'})$ does not depend on the procedure, and by the constrained efficiency of $\mathscr{B}$ this set is either $\{U^*_k, U^*_{k'}\}$ or empty, the partners must obtain the same utility outcome under $\mathscr{B}'$. Consequently, $\{d^*_{i_k}\}_{i_k,k}$, μ^*, U^* is an equilibrium also under $\mathscr{B}'$. Conversely, an equilibrium under $\mathscr{B}'$ is also an equilibrium under $\mathscr{B}$, establishing invariance. Therefore, to study the existence of equilibrium and the equilibrium matching patterns we can focus on a specific, analytically convenient bargaining procedure. We now turn to the analysis of such a procedure, $\mathscr{B}^*$.

3. The take-it-or-leave-it procedure $\mathscr{B}^*$

In the take-it-or-leave-it procedure $\mathscr{B}^*$, the p-agent proposes a sharing rule w, which is then accepted or rejected by the a-agent. The p-agent chooses w to maximize his utility $u_p(w, e_a, e_p, W)$, subject to the a-agent's participation constraint:

$$u_a(w, e_a, e_p) \geqslant \overline{U}_a, \tag{IR}$$

where $\overline{U}_a$ is the reservation utility of the a-agent. In addition, the contract has to satisfy the incentive and limited liability constraints (IC) and (LL) as expressed in Section 2. It is easily verified that the procedure $\mathscr{B}^*$ is constrained efficient.

For ease of exposition, hereafter we use the notation $w_1 \equiv w(X_1)$ and $\Delta w \equiv w(X_2) - w(X_1)$. From the (IC) constraints one can immediately derive the optimal efforts $e_p(\Delta w)$ and $e_a(\Delta w)$, and the probability of success $f(\Delta w)$ as continuous functions of Δw. Define

$$g_a(\Delta w) \equiv \Delta w f(\Delta w) - c\frac{e_a(\Delta w)^2}{2},$$

$$g_p(\Delta w) \equiv (1 - \Delta w) f(\Delta w) - c\frac{e_p(\Delta w)^2}{2},$$

$$g(\Delta w) \equiv g_a(\Delta w) + g_p(\Delta w),$$

where g_p is the gain to the p-agent, g_a the gain to the a-agent and g is the gain to the match, all net of wealth and the low state output. The following properties of g_a, g_p and g derive from the (IC) constraints and can easily be verified:

- $g_a(\Delta w)$ is strictly increasing.
- $g_p(\Delta w)$ and $g(\Delta w)$ are strictly concave, attaining maximums at Δw_p and Δw_g respectively, with $0 < \Delta w_p < \Delta w_g \leqslant 1$, with equality only for $\alpha = 1$.
- $g_a(\Delta w_g) \geqslant g_p(\Delta w_g)$ with equality only for $\alpha = \frac{1}{2}$.

Let $\{w_1^*(\overline{U}_a, W), \Delta w^*(\overline{U}_a, W)\}$ be the solution to the contracting problem expressed as a function of $\overline{U}_a$ and W. Let $S(\overline{U}_a, W) \equiv g(\Delta w^*(\overline{U}_a, W))$ be the value function, or (net) surplus computed at the optimum. After a bit of manipulation, using the properties of g_a, g_p and g, it is possible to establish the following properties of $S(\overline{U}_a, W)$ that will be used in characterizing the equilibrium.

Lemma 1. *Assume* $W + X_1 + g_a(1) \geqslant \overline{U}_a$. *Then the optimal surplus is well-defined and has the following properties*:

(a) *When* $\overline{U}_a < g_a(\Delta w_p)$, *only* (LL$_a$) *binds and* $S(\overline{U}_a, W) = g(\Delta w_p)$, *a constant independent of* $\overline{U}_a, W$.

(b) *When* $g_a(\Delta w_p) \leqslant \overline{U}_a < g_a(\Delta w_g)$, *both* (IR) *and* (LL$_a$) *bind*, $S(\overline{U}_a; W)$ *is concave in* $\overline{U}_a$, *with* $\frac{\partial S(\overline{U}_a; W)}{\partial \overline{U}_a} \in (0, 1)$, $\frac{\partial S(\overline{U}_a; W)}{\partial W} = 0$.

(c) *When* $g_a(\Delta w_g) \leqslant \overline{U}_a \leqslant W + X_1 + g_a(\Delta w_g)$, *only* (IR) *binds and* $S(\overline{U}_a, W) = g(\Delta w_g)$, *a constant independent of* $\overline{U}_a, W$.

(d) *When* $\overline{U}_a > W + X_1 + g_a(\Delta w_g)$, *then* (IR) *and* (LL$_p$) *bind*, $S(\overline{U}_a; W)$ *is concave in* $\overline{U}_a$, *with* $\frac{\partial S(\overline{U}_a, W)}{\partial \overline{U}_a} < 0 < \frac{\partial S(\overline{U}_a; W)}{\partial W}$.

Proof. Omitted.[7]

In case (a) we have 'efficiency wages', as $\overline{U}_a$ is so low that it is optimal to give the a-agent more than his outside option and $S(\overline{U}_a, W)$ is a constant. In case (b) the

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[7] For the proofs of the properties of g_a, g_p and g, and of Lemma 1, see [2]. It is easy to see that for the optimal contract, $\Delta w^*(\overline{U}_a, W) \in [0, 1]$. Thus, (LL$_a$) (resp. (LL$_p$)) can bind only in state X_1, if and only if $w_1^*(\overline{U}_a, W) = 0$ (resp., $= X_1 + W$).

optimal surplus is increasing in $\overline{U}_a$. Since (LL_a) and (IR) both bind, an increase in $\overline{U}_a$ implies that the a-agent must be paid more in the high state, improving his incentives to exert effort. While the p-agent's incentives will be so reduced, the net result on surplus will be non-negative as the a-agent's effort is less (and the p-agent's more) than what is optimal for maximizing surplus. Furthermore, since (LL_a) binds, one obtains from (IR) that $\Delta w^*(\overline{U}_a, W) = g_a^{-1}(\overline{U}_a)$, which does not depend on W, implying that $S(\overline{U}_a, W)$ does not depend on W in this case. In case (c) the optimal surplus $S(\overline{U}_a, W)$ is constant and equal to its maximum incentive compatible value. In case (d) $\overline{U}_a$ is so high that (LL_p) binds. Again, the a-agent has to be paid more in the high state when $\overline{U}_a$ rises, inducing him to exert more effort, but now his effort is too high compared to the optimal level, so that the net surplus falls as $\overline{U}_a$ rises. Moreover, since (LL_p) binds, one obtains from (IR) that $\Delta w^*(\overline{U}_a, W) = g_a^{-1}(\overline{U}_a - X_1 - W)$, implying that $S(\overline{U}_a, W)$ does depend on W in this case. In sum, Lemma 1 says that the optimal surplus as a function of $\overline{U}_a$, the a-agent's reservation utility, has a 'bell' shape; and it depends on W only when case (d) occurs. We will show in Lemma 2 that in any match that can be formed in equilibrium only case (b) or (c) can arise.

4. Characterization of equilibrium

In this section we demonstrate the existence of equilibrium, and also establish conditions that determine the matching patterns in equilibrium in terms of whether or not (LL_a) is binding for some type of a-agent. We introduce the following notation. For any k, we let $\mathbf{P}_k = \{j \in \mathbf{K} \mid \mu_j(P_k) > 0\}$ and $\mathbf{A}_k = \{j \in \mathbf{K} \mid \mu_j(A_k) > 0\}$ be, respectively, the set of types who are p- and a-agents of type k. Let $\mathbf{P} = \bigcup_k \mathbf{P}_k$ and $\mathbf{A} = \bigcup_k \mathbf{A}_k$ be the set of p-agents and of a-agents and let $\mathbf{I} = \{k \in \mathbf{K} \mid \mu_k(I) > 0\}$ be the set of self-employed types. Let $\mu(\mathbf{A}_k) = \sum_j \mu_j(A_k)$, $\mu(\mathbf{P}_k) = \sum_j \mu_j(P_k)$, $\mu(\mathbf{A}) = \sum_k \mu(\mathbf{A}_k)$, $\mu(\mathbf{P}) = \sum_k \mu(\mathbf{P}_k)$ and $\mu(\mathbf{I}) = \sum_k \mu_k(I)$. Essentially, two patterns will arise in equilibrium: segregation and job specialization.

Definition 2. For any economy θ, an equilibrium $\{d_{i_k}\}_{i_k,k}, \mu, U$ displays segregation if k matches with k' only if $k = k'$. It displays job specialization if $\mathbf{A} \cap \mathbf{P}$ contains at most one type, and strict job specialization if $\mathbf{A} \cap \mathbf{P} = \emptyset$. Job specialization is monotonic if $\max_k \mathbf{I} \cup \mathbf{P} \leqslant \min_k \mathbf{A}$ (with strict inequality if job specialization is strict).

In words, equilibrium displays segregation when equilibrium matches are homogeneous with respect to wealth: firms are formed of individuals all of the same wealth level. The economy is then separated into K juxtaposed, seemingly independent type-homogeneous economies. When there is job specialization, individuals are specialized in one job, except for possibly one type. When job specialization is monotonic, there is a threshold type separating the individuals into two classes, rich and poor, homogeneous by occupation, with the richer individuals taking the job for which incentives are more important.[8]

Let $k_0 = \min_k \mathbf{A}$ be the poorest *a*-agent (if **A** is non-empty). From Lemma 1, (LL_a) binds in a match with type k_0 if and only if

$$U_{k_0} < g_a(\Delta w_g), \tag{E*}$$

a condition on endogenous variables. First, we establish some central properties of the equilibrium, in terms of this condition, and then link this condition to the primitives of the model.

Lemma 2. *In any equilibrium*,

(a) (IR) *binds in the match* (j,k), *for all* $j,k \in \mathbf{K}$. *Further*, $\mathbf{I} = \emptyset$ *and* $\mu(\mathbf{A}) = \mu(\mathbf{P}) = \frac{1}{2}$.
(b) (LL_p) *does not bind in any match* (j,k), *for all* $j,k \in \mathbf{K}$.
(c) *There exists* $\gamma \geqslant 0$ *such that, for all* $k \in \mathbf{A}$ *and all* $j \in \mathbf{P}$,

$$U_j - W_j = \gamma = W_k + X_1 + S(U_k, W_j + W_k) - U_k \leqslant U_k - W_k. \tag{1}$$

(d) *For all* $k, k' \in \mathbf{K}$, *if* $k > k'$ *then* $U_k - W_k \geqslant U_{k'} - W_{k'}$.
(e) *If* (E*) *holds, then there is monotonic job specialization.*

Proof. See the appendix.

Lemma 2(a) is an immediate consequence of Lemma 1 and of conditions (ii) and (iii) in the definition of equilibrium. Lemma 2(b) is the key result as it shows that the surplus $S(U_k, W_j + W_k)$ in the match (j,k) in fact does not depend on the sum of the wealths of the partners $W_j + W_k$. Thus, we can simplify notation and denote the surplus simply as $S(U_k)$. A priori, the *a*-agent's reservation utility could be so high that he would have to receive a positive payment even in the low state, so that (LL_p) binds. However, this is at odds with the freedom all individuals have to choose occupations. If (LL_p) binds at the *equilibrium* match (j,k), then U_j is so low and U_k is so high that type *j* will do strictly better to match as a *p*-agent with his *own* type—he can guarantee himself at least the same share of surplus that he gets in the match (j,k) in addition to keeping $2W_j + X_1$. In fact, we show that (LL_p) cannot bind in any match (j,k), whether or not such a match is formed in equilibrium.

Since a richer individual can do everything a poorer individual can do, expected payoff net of wealth is weakly increasing in wealth. However, as the surplus in a match does not depend on who the *p*-agent is, all *p*-agents get the same net expected payoff γ. Since *a*-agents are in the more productive occupation, they get a weakly

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[8] Although job specialization could be monotonic and *decreasing* (where *poorer* individuals take the job for which incentives are more important), such a possibility cannot arise in equilibrium, as we show below, and we omit the qualifier 'increasing' in our definition of monotonic job specialization.

higher net expected payoff compared to p-agents. Indeed, when wealth effects matter for k_0, a match with a richer a-agent can achieve a *strictly* higher surplus. In such cases, any gain in surplus that is attained by a richer a-agent is captured entirely by him and richer individuals choose the more productive occupation, i.e., we obtain monotonic job specialization.

As richer a-agents capture the entire gain in surplus, p-agents are indifferent between matching with different types of a-agents, and Lemma 2(e) cannot generally be strengthened to yield a specific matching between $j \in \mathbf{P}$ and $k \in \mathbf{A}$. Once it is determined who the poorest a-agent is, and so, who the richest p-agent is, matching can occur between any p-agent and any a-agent. Hence, the equilibrium job allocation will not be completely determined even when (LL_a) is binding for k_0. Although the rich earn higher net surplus over wealth compared to the poor, poorer individuals prefer to match with the rich, receiving the wealth of the a-agent in exchange for giving up a higher portion of a larger surplus.

We now elaborate on Lemma 2 by linking condition (E^*) or its absence to conditions on the primitives of our economies, also demonstrating the existence of equilibrium. In order to do so, we let

$$\mathbf{R} \equiv \left\{ k \in \mathbf{K} \,\middle|\, W_k + \frac{X_1}{2} - \frac{g_a(\Delta w_g) - g_p(\Delta w_g)}{2} \geqslant 0 \right\}$$

be the set of rich (or unrestricted) types and let $\mathbf{R}^{\mathrm{c}}$ be its complement. When any type k lies in $\mathbf{R}$, equal division of the maximum incentive compatible surplus is achievable in any match with type k as an a-agent.[9]

Trivially, segregation is the only possible equilibrium if $K = 1$.[10] Accordingly, from now on we assume that $K \geqslant 2$. Let

$$k_{\mathrm{med}} \equiv \min \left\{ k \in \mathbf{K} \,\middle|\, \sum_{k' \leqslant k} \bar{\mu}_{k'} \geqslant \frac{1}{2} \right\},$$

be the median of the wealth distribution. The case $\sum_{k' \leqslant k_{\mathrm{med}}} \bar{\mu}_{k'} = 1/2$ is non-generic in the space of distributions that we consider. Although the essential features of equilibrium are unchanged in this knife-edge case, for clarity of exposition we present here our result only for the case $\sum_{k' \leqslant k_{\mathrm{med}}} \bar{\mu}_{k'} > 1/2$.

Proposition 1. *Assume* $K \geqslant 2$ *and* $\sum_{k' \leqslant k_{\mathrm{med}}} \bar{\mu}_{k'} > \frac{1}{2}$. *An equilibrium exists with the following properties.*

(a) *When* $k_{\mathrm{med}} \in \mathbf{R}^{\mathrm{c}}$, *then* (E^*) *holds. The equilibrium displays monotonic job specialization with* $k_0 = k_{\mathrm{med}}$. *It is unique in utilities and equilibrium utilities* U^* *satisfy* (1), *with*

$$\gamma = U^*_{k_{\mathrm{med}}} - W_{k_{\mathrm{med}}} = \tfrac{1}{2}[X_1 + S(U^*_{k_{\mathrm{med}}})] > 0.$$

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[9] The contract which achieves equal division of the maximum surplus (inclusive of X_1) has $\Delta w = \Delta w_g$ and w_1 equal to the left-hand side of the inequality in the definition of $\mathbf{R}$.

[10] In such a case, an equilibrium always exists, as the proof of Proposition 1 illustrates.

(b) *When* $k_{\text{med}} \in \mathbf{R}$, *then* (E*) *does not hold. The pattern of occupational choice satisfies* $\mathbf{A} \subset \mathbf{R}$, *with segregation being an equilibrium if and only if* $\mathbf{R}^{\text{c}} = \emptyset$. *The equilibrium is unique in utilities and equilibrium utilities* U^* *satisfy* (1), *with*

$$\gamma = \tfrac{1}{2}[X_1 + g(\Delta w_g)] > 0.$$

Proof. See the appendix.

Proposition 1 extends Lemma 2 by linking condition (E*) to the primitives of the economy. Since the measure of matches formed in equilibrium is equal to $\frac{1}{2}$, by market clearing we have $k_0 \leqslant k_{\text{med}}$. When k_{med} does not lie in $\mathbf{R}$, neither does k_0 and there is a positive measure of matches with a-agents who are not rich enough to achieve the maximum surplus and divide it equally. Then (LL$_a$) binds in these matches and (E*) holds. By Lemma 2 we obtain monotonic job specialization with $k_0 = k_{\text{med}}$. Since in equilibrium, individuals of type k_{med} must both be a- and p-agents, total surplus must be equally split between the two jobs for this type, determining γ.

When type k_{med} lies in $\mathbf{R}$, there is at least a measure $\frac{1}{2}$ of individuals in $\mathbf{R}$ who are rich enough to achieve the maximum incentive compatible surplus as a-agents, and divide it equally with the p-agents, without running afoul of limited liability constraints. A subset of these individuals will become a-agents in equilibrium, (LL$_a$) will not bind in any formed match and all matches will achieve the maximum surplus. Since more than half of the economy is rich, even poor individuals earn a high return from being a p-agent and the surplus attained in every match will be equally split. Although monotonic job specialization is still consistent with equilibrium, so are other matching patterns. As long as $\mathbf{R}^{\text{c}}$ is not empty however, some rich individuals must match with the poor.

When $\mathbf{R}^{\text{c}}$ is empty, all possible matches can achieve maximum surplus, wealth constraints are irrelevant and any matching pattern (in particular, segregation) is consistent with equilibrium. This necessary and sufficient condition for segregation to be an equilibrium is distribution-free (i.e., independent of $\bar{\mu}$). In the symmetric case of $\alpha = \frac{1}{2}$, $g_a(\Delta w_g) = g_p(\Delta w_g)$ so that $\mathbf{R} = \mathbf{K}$ and segregation is always an equilibrium. This case has been considered, among others, by Legros and Newman [10]. In contrast, when $\alpha > \frac{1}{2}$ there is an asymmetry in occupations. In such a case, if some types are not able to pay the transfers which are consistent with full surplus and equal division, they prefer to match with other types.[11]

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[11] For the knife-edge case where $\sum_{j \leqslant k_{\text{med}}} \bar{\mu}_j = \frac{1}{2}$, we get the essentially the same equilibria as in the generic case, but with more indeterminacy. If $k_{\text{med}} + 1 \in \mathbf{R}^{\text{c}}$, $\mathbf{A} \cap \mathbf{P} = \emptyset$ and the equilibrium displays (strict) monotonic job specialization. There exists a continuum of possible equilibrium utility profiles. These can be obtained by treating any wealth level between $W_{k_{\text{med}}}$ and $W_{k_{\text{med}}+1}$ as indifferent between occupations. If $k_{\text{med}} + 1 \in \mathbf{R}$ but $k_{\text{med}} \in \mathbf{R}^{\text{c}}$, we get the same occupational pattern as in Proposition 1(b), but unequal division of maximum surplus is possible. If $k_{\text{med}} \in \mathbf{R}$, equal division of maximum surplus with segregation is obtained if and only if $\mathbf{R}^{\text{c}} = \emptyset$.

5. Changes in the wealth distribution

Proposition 1 implies that the equilibrium payoff U_k and the net payoff $U_k - W_k$ generally depend on the distribution of wealth. We now characterize how changes in the distribution of wealth will alter the equilibrium payoffs of different types and the returns to different occupations. We compare economies that differ in their median, whether or not they have the same total wealth.

Proposition 2 (Trickle down effect). *For any two economies* θ^1 *and* θ^2 *with* $W_{k^2_{\text{med}}} > W_{k^1_{\text{med}}}$ *and* $k^1_{\text{med}} \in \mathbf{R}^c$, *there exists* $\widehat{W} \in (W_{k^1_{\text{med}}}, W_{k^2_{\text{med}}})$ *such that, in equilibrium, types with wealth level less than* $\widehat{W}$ *are strictly better off and types with wealth level greater than* $\widehat{W}$ *are strictly worse off in the economy* θ^2 (*and* $\widehat{W}$ *is indifferent*).

Proof. See the appendix.

When the wealth level of the median goes up (through either changes in $\bar{\mu}$ or changes in the W_k's), the net payoff to the median increases. As a result, the net payoff to all p-agents increases. Since the total measure of matches is fixed while lower wealth levels are less numerous, market clearing implies some a-agent types must switch to become p-agents in the new economy. For types who stay on the same side of the market, a-agents lose and p-agents gain. We call this the trickle down effect of a change in the distribution.[12] Fig. 1 illustrates the proposition.

In Fig. 1, we plot the net payoff $U_k - W_k$ as a function of W_k for economies with different median wealth levels $W_{k_{\text{med}}}$. For the parameters chosen, $\mathbf{R} = \{k | W_k \geqslant 0.125\}$.[13] The increasing curve that lies below all the other curves traces the net payoff of different median types. The other two curves correspond to the equilibrium distribution of net payoffs in two economies 1 and 2, where $W^1_{k_{\text{med}}} = 0.025$ and $W^2_{k_{\text{med}}} = 0.05$. In each economy, there is monotonic job specialization with $\gamma^1 = 0.1175$ and $\gamma^2 = 0.1213$. All types with wealth less than (respectively, more than) 0.03403 are better off (resp., worse off) in economy 2. In economies where $W_{k_{\text{med}}} \geqslant 0.125$, monotonic job specialization does not necessarily obtain, although all a-agents are still above the median. In such cases, all matches earn the maximum surplus 0.25 which is equally divided. Finally, segregation is an equilibrium only for those economies where $W_1 \geqslant 0.125$.

With respect to the effect of the median on total surplus, $\sum_{k \in \mathbf{K}} (U_k - W_k)\bar{\mu}_k$, notice that this can also be written as

$$S(U_{k_{\text{med}}})\left(\frac{1}{2} - \sum_{k > k_{\text{med}}} \bar{\mu}_k\right) + \sum_{k > k_{\text{med}}} S(U_k)\bar{\mu}_k.$$

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[12] This effect was also noted in [9].

[13] The parameters are $\alpha = 1$, $X_1 = 0$ and $c = 2$. For these parameters, the maximum net surplus is equal to 0.25.

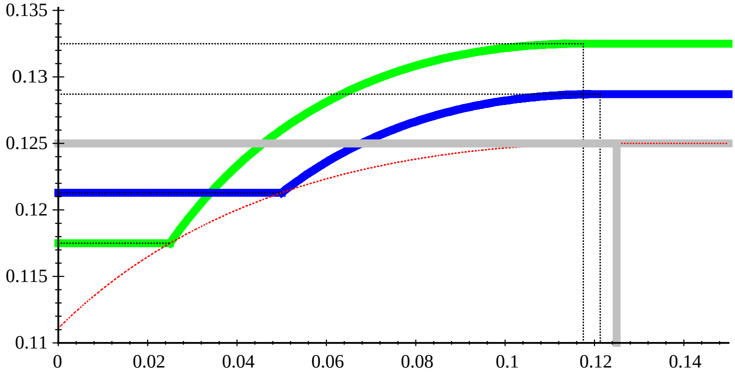

Fig. 1. The trickle down effect.

From Proposition 2, for any differential increase in the wealth of the median class, $U_{k_{\text{med}}}$ rises but U_k falls for all $k > k_{\text{med}}$. As long as $k_{\text{med}} \in \mathbf{R}^c$ before the wealth increase, this leads to a (weak) fall in $S(U_k)$ for all $k > k_{\text{med}}$ and a strict rise in $S(U_{k_{\text{med}}})$. The overall effect on total surplus is generally ambiguous.

Finally, consider the effect of changes in wealth inequality on total surplus. From the concavity of $S(U)$ (see Lemma 1) and from Eq. (1) it follows that the net payoff $U_k - W_k$ is a concave function of wealth for $W_k > W_{k_{\text{med}}}$. Standard results on mean preserving spreads and risk aversion (see [15]) then apply to show that a budget-balanced redistribution policy that leads to a reduction in the inequality in the distribution of wealth for types $k > k_{\text{med}}$ is going to increase total surplus, provided that the median wealth level is unaffected. However, the effect on total surplus (and its distribution) of more general redistribution policies that may change the median is ambiguous.

Appendix

Proof of Lemma 2. (a) Suppose that for some (j, k), (IR) does not bind. From Lemma 1, $U_k < g_a(\Delta w_p) = U_a(\mathcal{B}^*, W_j + W_k, U_j, U_k)$, and individual k gets strictly more than U_k, a contradiction with equilibrium. Furthermore, if $\mathbf{I} \neq \emptyset$, then there exists k with $U_k = W_k$. But in the match (k, k), the p-agent earns $W_k + X_1 + S(W_k, W_k + W_k) > W_k$, a contradiction with equilibrium. Thus, $\mathbf{I} = \emptyset$ and from market clearing it follows that $\mu(\mathbf{A}) = \mu(\mathbf{P}) = \frac{1}{2}$.

(b), (c) We prove parts (b) and (c) in steps, as follows. In Step 1, we show that in any match (j, k) that is realized in equilibrium, (LL$_p$) cannot bind. In Step 2, we show that (LL$_p$) does not bind in any match (j, k) with $k \in \mathbf{A}$, realized or not. In Step 3, we use this to establish (c) and also complete (b) by showing that (LL$_p$) does not bind in any match (j, k).

Step 1: If not, there exists $k \in \mathbf{A}$ and $j \in \mathbf{P}_k$ such that $U_k > W_j + W_k + X_1 + g_a(\Delta w_g)$, from Lemma 1, so that from the properties of g_a and g_p,

$$U_j = g_p(\Delta w^*(U_k, W_j + W_k)) < g_p(\Delta w_g) \leqslant g_a(\Delta w_g).$$

Then, in the match (j,j), (LL_a) binds and the type-j p-agent earns

$$2W_j + X_1 + g_p(\Delta w^*(U_j, W_j + W_j)) > g_p(\Delta w_g) > U_j,$$

by Lemma 1, a contradiction with equilibrium.

Step 2: We show that for any $k \in \mathbf{A}$, (LL_p) does not bind in the match $(1,k)$, which implies that it does not bind in the match (j,k) for any j.

Suppose not. From Lemma 1 there exists $k \in \mathbf{A}$ such that

$$U_k > W_1 + W_k + X_1 + g_a(\Delta w_g). \tag{A.1}$$

Pick any $k' \in \mathbf{P}_k$ and note from Step 1 that $k' > 1$. Since the match (k',k) achieves at most $g(\Delta w_g)$, we have

$$U_{k'} \leqslant W_{k'} + W_k + X_1 + g(\Delta w_g) - U_k < W_{k'} - W_1 + g_p(\Delta w_g) \tag{A.2}$$

using (A.1). Thus, from the properties of g_a and g_p, (LL_p)does not bind in the match $(1,k')$. Further, as type 1's equilibrium payoff is at least as high as he obtains in the match $(1,k')$, we must have, using $k' > 1$,

$$U_1 \geqslant W_1 + W_{k'} + X_1 + g_p(\Delta w^*(U_{k'}, W_1 + W_{k'})) > g_p(\Delta w_g)$$

so that from (A.2) we obtain

$$U_{k'} < W_{k'} - W_1 + U_1 \tag{A.3}$$

We now obtain a contradiction by showing that type 1 cannot belong to $\mathbf{P}$ or $\mathbf{A}$. If $1 \in \mathbf{P}$, with $j' \in \mathbf{A}_1$, then (LL_p) does not bind in the equilibrium match $(1,j')$ by step 1, so that it also does not bind in the match (k',j'). Then from Lemma 1 the surplus in the matches (k',j') and $(1,j')$ are equal, so that in the match (k',j'), type k' gets $W_{k'} - W_1 + U_1 > U_{k'}$, using (A.3), a contradiction with equilibrium. So suppose that $1 \in \mathbf{A}$, with $j' \in \mathbf{P}_1$. Since (LL_p) does not bind in the match $(1,k')$ (by (A.2)), it does not bind in the match (j',k'). As type j''s equilibrium payoff is at least as high as the payoff in the match (j',k'), then

$$\begin{aligned} U_{j'} &= W_{j'} + W_1 + X_1 + S(U_1, W_{j'} + W_1) - U_1 \\ &\geqslant W_{j'} + W_{k'} + X_1 + S(U_{k'}, W_{j'} + W_{k'}) - U_{k'} \end{aligned}$$

Rearranging,

$$[U_{k'} - S(U_{k'}, W_{j'} + W_{k'})] - [U_1 - S(U_1, W_{j'} + W_1)] \geqslant (W_{k'} - W_1) > 0 \tag{A.4}$$

Since (LL_p) does not bind in either the match (j',k') or the equilibrium match $(j',1)$, from Lemma 1, neither of the two surplus functions above depend on the wealth levels. Further, the derivative of the surplus function with respect to U is in $[0,1)$ so that applying the Mean Value Theorem to the left-hand side of (A.4) we obtain $U_{k'} - U_1 \geqslant W_{k'} - W_1$, a contradiction with (A.3).

Step 3: For any j and $k, k' \in \mathbf{A}$, j prefers P_k to $P_{k'}$ if and only if, using part (a),

$$W_k + S(U_k, W_j + W_k) - U_k \geqslant W_{k'} + S(U_{k'}, W_j + W_{k'}) - U_{k'}.$$

From Step 2 and Lemma 1, neither $S(U_k, W_j + W_k)$ nor $S(U_{k'}, W_j + W_{k'})$ depend on W_j, W_k or $W_{k'}$ and we can write them simply as $S(U_k)$ and $S(U_{k'})$ for $k, k' \in \mathbf{A}$. The ranking on $\mathbf{A}$ is thus common to all $j \in \mathbf{P}$, so that for $k, k' \in \mathbf{A}$ the expression above must hold as equality and $U_j - W_j$ must be a nonnegative constant γ. Further, since any $k \in \mathbf{A}$ must weakly prefer being an a-agent to matching with his own type as a p-agent, using the second equality in (1) we obtain $\gamma \leqslant U_k - W_k$ for all $k \in \mathbf{A}$. This completes part (c).

Since for any $k \in \mathbf{P}$ and $k' \in \mathbf{A}$, $U_k = W_k + \gamma \leqslant W_k + U_{k'} - W_{k'}$, using Step 3 and Lemma 1 we immediately obtain that (LL_p) does not bind in the match $(1, k)$. Thus, (LL_p) does not bind in *any* match (j, k) and allows us to write the surplus simply as $S(U_k)$, as it does not depend directly on the wealth of the partners.

(d) For $k, k' \in \mathbf{P}$, or for $k \in \mathbf{A}$ and $k' \in \mathbf{P}$ with $k > k'$, we immediately obtain $U_k - W_k \geqslant U_{k'} - W_{k'}$ from (1). So suppose that $k' \in \mathbf{A}$ and pick any k with $k > k'$. Pick any $j \in \mathbf{P}_{k'}$. Since j must weakly prefer matching with k' to matching as a p-agent with k, and using part (b) we must have $W_{k'} + S(U_{k'}) - U_{k'} \geqslant W_k + S(U_k) - U_k$. The properties of the surplus function in Lemma 1 and the Mean Value Theorem again yield the desired inequality.

(e) From the definition of k_0, $\mathbf{P}_{k_0} \neq \emptyset$ and $k < k_0$ implies $k \notin \mathbf{A}$ so that $k \in \mathbf{P}$. From part (d) $k > k_0$ implies $U_k > U_{k_0}$. Condition (E*), Lemma 1 and part (b) then imply that, in a match (j, k) for any $j \in \mathbf{K}$, $S(U_k) > S(U_{k_0})$. If $k \in \mathbf{P}$, then from part (c) (i.e., Eq. (1)) $U_k - W_k = \gamma$. Then, any j strictly prefers the match (j, k) to (j, k_0), so that we have $\mathbf{P}_{k_0} = \emptyset$, a contradiction. Hence, $k > k_0$ implies $k \in \mathbf{A}$, or monotonic job specialization. □

Proof of Proposition 1. (a) We divide the proof in two steps. In Step 1, we establish that if $k_{\mathrm{med}} \in \mathbf{R}^{\mathrm{c}}$ then condition (E*) holds. In Step 2 we show that if (E*) holds then $k_0 = k_{\mathrm{med}}$, $\gamma = U_{k_{\mathrm{med}}} - W_{k_{\mathrm{med}}}$ where $U_{k_{\mathrm{med}}}$ is the (unique) solution to (1) for $j = k = k_{\mathrm{med}}$.

Step 1: Suppose that $k_{\mathrm{med}} \in \mathbf{R}^{\mathrm{c}}$ but (E*) does not hold, i.e., (LL_a) does not bind for k_0. From Lemma 2(d) it is seen that for $k > k'$ we must have $U_k > U_{k'}$ so that (LL_a) does not bind in any match, from Lemma 1. By Lemma 2(c) the maximum surplus $g(\Delta w_g)$ is achieved in every realized match and

$$U_k - W_k \equiv \gamma_a \geqslant \gamma = U_j - W_j$$

for all $k \in \mathbf{A}$ and all $j \in \mathbf{P}$, with $\gamma_a + \gamma = X_1 + g(\Delta w_g)$.

Since $k_{\mathrm{med}} \in \mathbf{R}^{\mathrm{c}}$ and $\sum_{k \leqslant k_{\mathrm{med}}} \bar{\mu}_k > 1/2$, we then have $\sum_{k \in \mathbf{R}} \bar{\mu}_k < 1/2$. Since $\mu(\mathbf{A}) = \frac{1}{2}$, there must then exist $k' \in \mathbf{A}$ with $k' \leqslant k_{\mathrm{med}}$ so that $k' \in \mathbf{R}^{\mathrm{c}}$.

If $\gamma_a = \gamma = \frac{X_1 + g(\Delta w_g)}{2}$, since $k' \in \mathbf{R}^{\mathrm{c}}$, we must have $U_{k'} = W_{k'} + \gamma_a < g_a(\Delta w_g)$, contradicting the fact that (LL_a) does not bind in any match. On the other hand, if $\gamma_a > \gamma$, no type is indifferent between being an a-agent and being a p-agent, so we must have $\mathbf{A} \cap \mathbf{P} = \emptyset$. Since $\sum_{k' \leqslant k_{\text{med}}} \bar{\mu}_{k'} > 1/2$ and $\mu(\mathbf{A}) = \frac{1}{2}$, this implies that $k_{\text{med}} \notin \mathbf{A}$ and $k' < k_{\text{med}}$. But then $U_{k'} - W_{k'} = \gamma_a > \gamma = U_{k_{\text{med}}} - W_{k_{\text{med}}}$, in contradiction with Lemma 2(d). Thus (E^*) must hold.

Step 2: Suppose condition (E^*) holds. From Lemma 2(e) we obtain monotonic job specialization. Since $\mu(\mathbf{A}) = \mu(\mathbf{P}) = \frac{1}{2}$ and $\sum_{k' \leqslant k_{\text{med}}} \bar{\mu}_{k'} > 1/2$, from the definition of k_{med} we must have $k_0 = k_{\text{med}}$ with $k_{\text{med}} \in \mathbf{A} \cap \mathbf{P}$. Since k_{med} is indifferent between occupations, Eq. (1) must hold with $j = k = k_{\text{med}}$. In particular, let

$$\Phi(U_{k_{\text{med}}}) = \frac{X_1 + S(U_{k_{\text{med}}})}{2} - U_{k_{\text{med}}} + W_{k_{\text{med}}}.$$

At $U_{k_{\text{med}}} = g_a(\Delta w_p)$, we have $\Phi(g_a(\Delta w_p)) > 0$ using the fact that $g_p(\Delta w_p) > g_a(\Delta w_p)$. Also, at $U_{k_{\text{med}}} = g_a(\Delta w_g)$, we have $\Phi(g_a(\Delta w_g)) < 0$, since $k_{\text{med}} \in \mathbf{R}^{\text{c}}$. By the Intermediate Value Theorem, a $U_{k_{\text{med}}}$ such that $\Phi(U_{k_{\text{med}}}) = 0$ exists in the range $(g_a(\Delta w_p), g_a(\Delta w_g))$ and is unique by the monotonicity of the surplus in U. Moreover, (LL_a) binds in a match with type k_{med} of a-agent. From the properties of g_a and g_p, $U_{k_{\text{med}}} > g_p(\Delta w_g)$. Using this observation, another application of the Intermediate Value Theorem determines U_k in the range between $U_{k_{\text{med}}}$ and $W_{k_{\text{med}}} + W_k + X_1 + g_a(\Delta w_g)$ from Eq. (1), for $k > k_{\text{med}}$. Monotonicity of $S(U) - U$ again implies the uniqueness of U_k. Finally, using (1) again, we immediately determine a unique U_k for $k < k_{\text{med}}$.

(b) Suppose that $k_{\text{med}} \in \mathbf{R}$. If ($\text{E}^*$) holds, then monotonic job specialization obtains from Lemma 2(e) and we must have $k_0 = k_{\text{med}} \in \mathbf{A} \cap \mathbf{P}$, as in part (a), Step 2. However, since $k_{\text{med}} \in \mathbf{R}$, $\Phi(U)$ will not have a solution for $U_{k_{\text{med}}}$ in the range $(g_a(\Delta w_p), g_a(\Delta w_g))$, so that ($\text{LL}_a$) will not bind in the match with an a-agent of type $k_{\text{med}} = k_0$, a contradiction with (E^*). Thus (E^*) cannot hold so that (LL_a) cannot bind in any match, by the monotonicity of U_k in k obtained from Lemma 2(d) Each realized match must earn the maximum surplus $g(\Delta w_g)$, so that from (1) we obtain

$$U_k - W_k \equiv \gamma_a \geqslant \gamma = U_j - W_j$$

for all $k \in \mathbf{A}$ and all $j \in \mathbf{P}$, with $\gamma_a + \gamma = X_1 + g(\Delta w_g)$.

Since $k_{\text{med}} \in \mathbf{R}$ and $\sum_{k' \leqslant k_{\text{med}}} \bar{\mu}_{k'} > 1/2$, we then have $\sum_{k \in \mathbf{R}^{\text{c}}} \bar{\mu}_k < 1/2$. Since $\mu(\mathbf{P}) = \frac{1}{2}$, there must then exist $k' \in \mathbf{P}$ with $k' \geqslant k_{\text{med}}$ so that $k' \in \mathbf{R}$.

If $\gamma_a > \gamma$, then $\mathbf{A} \cap \mathbf{P} = \emptyset$. Since $\sum_{k' \leqslant k_{\text{med}}} \bar{\mu}_{k'} > 1/2$, we must then have $k_{\text{med}} \notin \mathbf{P}$ and $k' > k_{\text{med}}$. But then $U_{k'} - W_{k'} = \gamma < \gamma_a = U_{k_{\text{med}}} - W_{k_{\text{med}}}$ in contradiction with Lemma 2(d). Thus $\gamma_a = \gamma = \frac{X_1 + g(\Delta w_g)}{2} \equiv U_k^{**} - W_k$ is obtained for every k.[14] Furthermore, if $\mathbf{R}^{\text{c}} \neq \emptyset$, then for $k \in \mathbf{R}^{\text{c}}$, we have $U_k^{**} = W_k + \frac{X_1 + g(\Delta w_g)}{2} < g_a(\Delta w_g)$ so that $k \in \mathbf{P}$, as otherwise (LL_a) would bind in the match with type k of a-agent, in contradiction with the fact that each realized match earns maximum surplus. Hence, $\mathbf{A} \subset \mathbf{R}$, i.e., $\mathbf{R}^{\text{c}} \subset \mathbf{P}$. This shows that the equilibrium displays segregation only if $\mathbf{R}^{\text{c}} = \emptyset$. In the

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[14] This, together with the zero we found for $\Phi(U)$ establishes the existence of equilibrium also for the case where $K = 1$.

other direction, if $\mathbf{R}^c = \emptyset$, then $1 \in \mathbf{R}$ and the two inequalities

$$U_k^{**} - 2W_k - X_1 \leqslant g_a(\Delta w_g) \leqslant U_k^{**},$$

can be satisfied strictly at all $k \in \mathbf{K}$. Then the maximum surplus $g(\Delta w_g)$ can be attained by all pairs (k, k), with total payoff equal to U_k^{**} for each partner. Since the net payoff $U_k^{**} - W_k$ is independent of k, it is then straightforward to check that it is weakly optimal for any type to be segregated, and an equilibrium with segregation exists. This completes the proof. □

Proof of Proposition 2. Consider two economies θ^1 and θ^2, with medians $W_{k^1_{\text{med}}}$ and $W_{k^2_{\text{med}}}$ respectively, and $W_{k^1_{\text{med}}} < W_{k^2_{\text{med}}}$. We denote by U^1, U^2 their equilibrium vectors. Let $\Phi(U_{k_{\text{med}}}) = 0$ define implicitly $U_{k_{\text{med}}} = \phi(W_{k_{\text{med}}})$, the utility of the median type as a function of his wealth. Note that $\phi'(W_{k_{\text{med}}}) = \frac{1}{1-S'/2} \geqslant 1$. Since $k^1_{\text{med}} \in \mathbf{R}^c$, we must in fact have $\phi'(W_{k^1_{\text{med}}}) > 1$ so that $U^2_{k^2_{\text{med}}} - W_{k^2_{\text{med}}} > U^1_{k^1_{\text{med}}} - W_{k^1_{\text{med}}}$, or $\gamma^2 > \gamma^1$. Since, $U_k^j - W_k = \gamma^j$ for all $W_k \leqslant W_{k^1_{\text{med}}}$, $j = 1, 2$, we get $U_k^1 < U_k^2$ for all $k \leqslant k^1_{\text{med}}$.

For $k \geqslant k^2_{\text{med}}$, (1) for $j = 1, 2$ implies that

$$W_k + X_1 + S(U_k^j) - U_k^j - \gamma^j = 0 \tag{A.5}$$

Since $\gamma^2 > \gamma^1$ we have $S(U_k^2) - U_k^2 > S(U_k^1) - U_k^1$ or $U_k^2 < U_k^1$, as $U - S(U)$ is increasing in U.

Let $U_k^1 = \varphi(W_k, \gamma^1)$ be the solution to (A.5). The difference $U_k^2 - U_k^1$ for $W_k \in [W_{k^1_{\text{med}}}, W_{k^2_{\text{med}}}]$ is equal to $W_k + \gamma^2 - \varphi(W_k, \gamma^1)$. From the arguments above, this is positive at $W_k = W_{k^1_{\text{med}}}$ and negative at $W_k = W_{k^2_{\text{med}}}$. Furthermore, $\frac{\partial}{\partial W_k}\varphi(W_k, \gamma^1) = \frac{1}{1-S'} \geqslant 1$ with a strict inequality at $W_k = W_{k^1_{\text{med}}}$, as $k^1_{\text{med}} \in \mathbf{R}^c$. By the Intermediate Value Theorem, there exists a unique $\widehat{W} \in (W_{k^1_{\text{med}}}, W_{k^2_{\text{med}}})$ such that $U_k^1 > U_k^2$ if $W_k < \widehat{W}$ and $U_k^1 < U_k^2$ if $W_k > \widehat{W}$. □

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